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An Analysis of Turbulent Flow Around a NACA4412 Airfoil by Using a Segregated Finite Element Method

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A turbulent flow around a NACA4412 airfoil is simulated by a segregated finite element method based on the SIMPLE algorithm and the low Reynolds number $k-\omega$ turbulence model. The original $k-\omega$ model and a modified version of the $k-\omega$ model (shear stress transport model) are adopted, for which grid independent solutions are obtained, respectively. From the present numerical experiment, it has been shown that the segregated finite element method with the $k-\omega$ turbulence model can predict the turbulent flow leading to separation satisfactorily with apparently reduced memories compared with the mixed integrated formulation. It is also recommended that for the analysis of external flows a modified $k-\omega$ model should be used instead of the original $k-\omega$ model, which combines the features of both the standard $k-\varepsilon$ model and the original $k-\omega$ model.

Key Words: Segregated FEM, NACA4412 Airfoil, Shear Stress Transport Model

1. Introduction

In spite of the ability to handle complex geometries easily, the finite element method was not as widely used as it should have been for solving the incompressible. Navier-Stokes equations because large memory and computing time are usually required. To overcome these drawbacks, many researchers have suggested cost-effective algorithms which satisfy the continuity constraint from the Poisson type pressure equations and solve the Navier-Stokes equations in a sequential way with smaller memory than the velocity-pressure integrated method. In the present study, the segregated finite element algorithm developed by Choi & Yoo (1994) is used, which is based on the equal-order finite element SIMPLE algorithm proposed by Rice & Schnipke (1986) and adopts the SUPG (streamline upwind Petrov-Galerkin) method as the upwind scheme. The segregated formulation requires only about one-ninth memory compared with the mixed formulation because u, v and p are solved separately.

In the present study, the low Reynolds number $k-\omega$ turbulence model is used to simulate the turbulent flow around a NACA4412 airfoil. Both the original $k-\omega$ model proposed by Wilcox (1988) and the SST (shear stress transport) model suggested by Menter (1994) are to be tested. Menter showed that the SST model predicted the external flows satisfactorily and used the SST model for the analysis of the turbulent flow around a NACA4412 airfoil at the maximum lift angle of 13.87° and the Reynolds number based on the chord length $Re=1.52\times10^{6}$. He argued that his result agreed well with the experimental result of Coles & Wadcock (1979) in terms of velocity profiles. However, this was not supported by any grid tests. Furthermore, a later study of Wadcock (1987) showed that the maximum lift angle was not13.87° but 12° implying that in their earlier experiment (Coles & Wadcock, 1979), the freestream flow was not parallel to the wind tunnel wall. In the present

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study, we test a code developed on the basis of the segregated finite element method (Choi & Yoo, 1994) and the SST model (Menter, 1994) against the experimental result of Hastings & Williams (1987) at the maximum lift angle of 12.15°, where grid independent solutions for the two models are to be obtained.

2. Numerical Method

2.1 Governing equations

Two of the modified versions of the $k-\omega$ model proposed by Menter (1994) are the baseline model and the SST (shear stress transport) model. The baseline model utilizes the original $k-\omega$ model in the inner region of the boundary layer and switches to the standard $k-\varepsilon$ model in the outer region and in free shear flows to avoid the original $k-\omega$ model's strong freestream sensitivity. The SST model uses a different eddy-viscosity definition and empirical constants in the baseline model, which is known to lead to improvements in the prediction of adverse pressure gradient flows.

The governing equations for two-dimensional incompressible flows are the continuity equation, the Navier-Stokes equation and the transport equations for the turbulent quantities represented by the low Reynolds number $k-\omega$ model:

$$
\begin{aligned}
& u_{i,i}=0, \\
& \rho u_j\frac{\partial u_i}{\partial x_j}=-p_{,i}+\frac{\partial}{\partial x_j}\left(\mu_e\frac{\partial u_i}{\partial x_j}\right), \qquad (1)\\
& \rho u_j\frac{\partial k}{\partial x_j}=\tau_{ij}\frac{\partial u_i}{\partial x_j}-\beta^*\rho k\omega+\frac{\partial}{\partial x_j}\left[(\mu+\sigma_k\mu_t)\frac{\partial k}{\partial x_j}\right], \\
& \rho u_j\frac{\partial \omega}{\partial x_j}=\frac{\gamma}{\nu_t}\tau_{ij}\frac{\partial u_i}{\partial x_j}-\beta\rho\omega^2+\frac{\partial}{\partial x_j}\left[(\mu+\sigma_\omega\mu_t)\frac{\partial \omega}{\partial x_j}\right]+2(1-F_1)\rho\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j},
\end{aligned}
$$

where $\mu_e=\mu+\mu_t$, $\mu_t=\rho k/\omega$, $\beta^*=0.09$, k is the turbulent kinetic energy, $\omega=\varepsilon/k$ is the specific dissipation rate, F_1 is a parameter dependent on the flow characteristics, and the ω-equation is written in the form of the baseline model (Menter, 1994). The set of empirical constants $\Phi=(\sigma_k, \sigma_\omega, \beta, \gamma)$ used in the baseline model are calculated from two sets of constants $\Phi_1$and Φ_2 as follows:

$$\Phi=F_1\Phi_1+(1-F_1)\Phi_2, \qquad (2)$$

where the set of constants Φ_1 are from the original $k-\omega$ model such that

$$
\begin{aligned}
& \sigma_{k1}=0.5,\ \sigma_{\omega 1}=0.5,\ \beta_1=0.075,\ \beta^*=0.09, \\
& \gamma_1=\beta_1/\beta^*-\frac{\sigma_{\omega 1}\kappa^2}{\sqrt{\beta^*}} \qquad (3)
\end{aligned}
$$

and the set of constants $k-\omega$ are from the standard $k-\varepsilon$ model such that

$$
\begin{aligned}
& \sigma_{k2}=1.0,\ \sigma_{\omega 2}=1.856,\ \beta_2=0.0828,\ \beta^*=0.09, \\
& \gamma_2=\beta_2/\beta^*-\frac{\sigma_{\omega 2}\kappa^2}{\sqrt{\beta^*}} \qquad (4)
\end{aligned}
$$

It is noted that if $F_1=1$, then we recover the original $k-\omega$ model. The empirical constants Φ used in the SST model are identical to those of the baseline model except that the constant σ_{k_1} has to be changed to $\sigma_{k_1}=0.85$ and ν_t is defined somewhat differently (Menter, 1994). In the present study, the original $k-\omega$ model and the SST model are to be considered.

2.2 Boundary conditions

Boundary conditions for the freestream are given as follows:

$$
\begin{aligned}
& \omega_\infty=\frac{U_\infty}{L},\ \nu_{t\infty}=10^{-2}\nu_\infty=\nu_{t\infty}\omega_\infty, \qquad (5)\\
& u=U_\infty,\ v=0,
\end{aligned}
$$

where L is the chord length of the airfoil. Boundary conditions at the solid surface are given as follows:

$$\omega=10\frac{6\nu}{\beta_1(\Delta y_1)^2},\ u=v=k=0, \qquad (6)$$

where Δy_1 is the distance from the wall to the nearest point. In the present study, the dimensionless distance from the wall, Δy_1^+ is taken to be less than 3. From a computational point of view, the $k-\omega$ model is easy to implement because it does not require any damping functions in the viscous sublayer and it adopts simple Dirichlet boundary conditions for k and ω at the boundaries.

2.3 Solution algorithm

The solution algorithm is the SUPG finite element method based on the SIMPLE algorithm, in which the discretized equations for u, v, k and ω are derived by the weighted residual method with SUPG and the Poisson type pressure equation is obtained by the continuity constraint.

Table 1 Four unstructured meshes used to obtain grid independent solution.

Mesh	Total No. of nodes	Surface nodes	Minimum spacing at trailing edge	Minimum spacing at leading edge	Stretching ratio normal to the wall
Grid I	44010	450	4.5×10^{-4}	3.0×10^{-4}	1.05
Grid II	35913	400	5.0×10^{-4}	3.8×10^{-4}	1.1
Grid III	30542	360	5.2×10^{-4}	4.5×10^{-4}	1.2
Grid IV	25756	200	5.8×10^{-4}	20.0×10^{-4}	1.2

Assembling element matrices, the global discretized equation is obtained and solved by the biconjugate gradient stabilized method with diagonal preconditioning. A detailed procedure for the derivation of the pressure equation can be found in the studies of Choi & Yoo (1994) and Rice & Schnipke (1986). The symmetric pressure equation is solved by ICCG (incomplete Cholesky conjugate gradient).

2.4 Grid system

In the present study, a hybrid grid system is imposed on the computational domain, which is a combination of structured and unstructured grids. A structured grid is established in the viscous

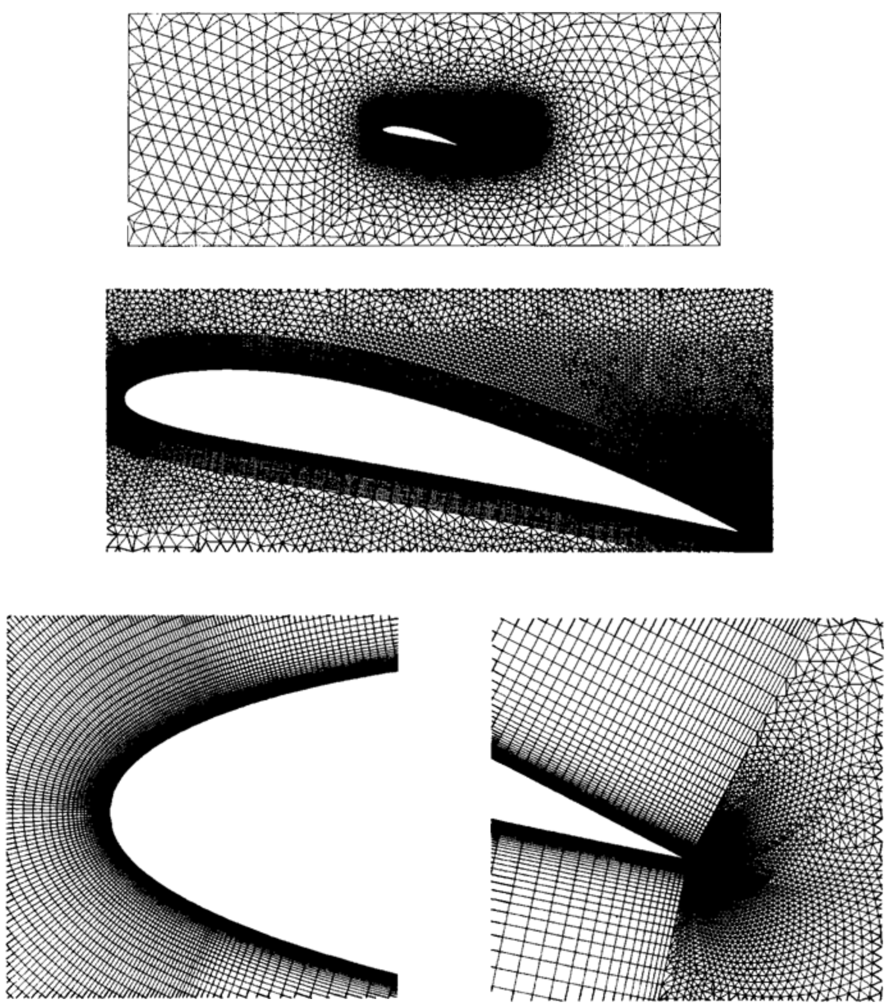
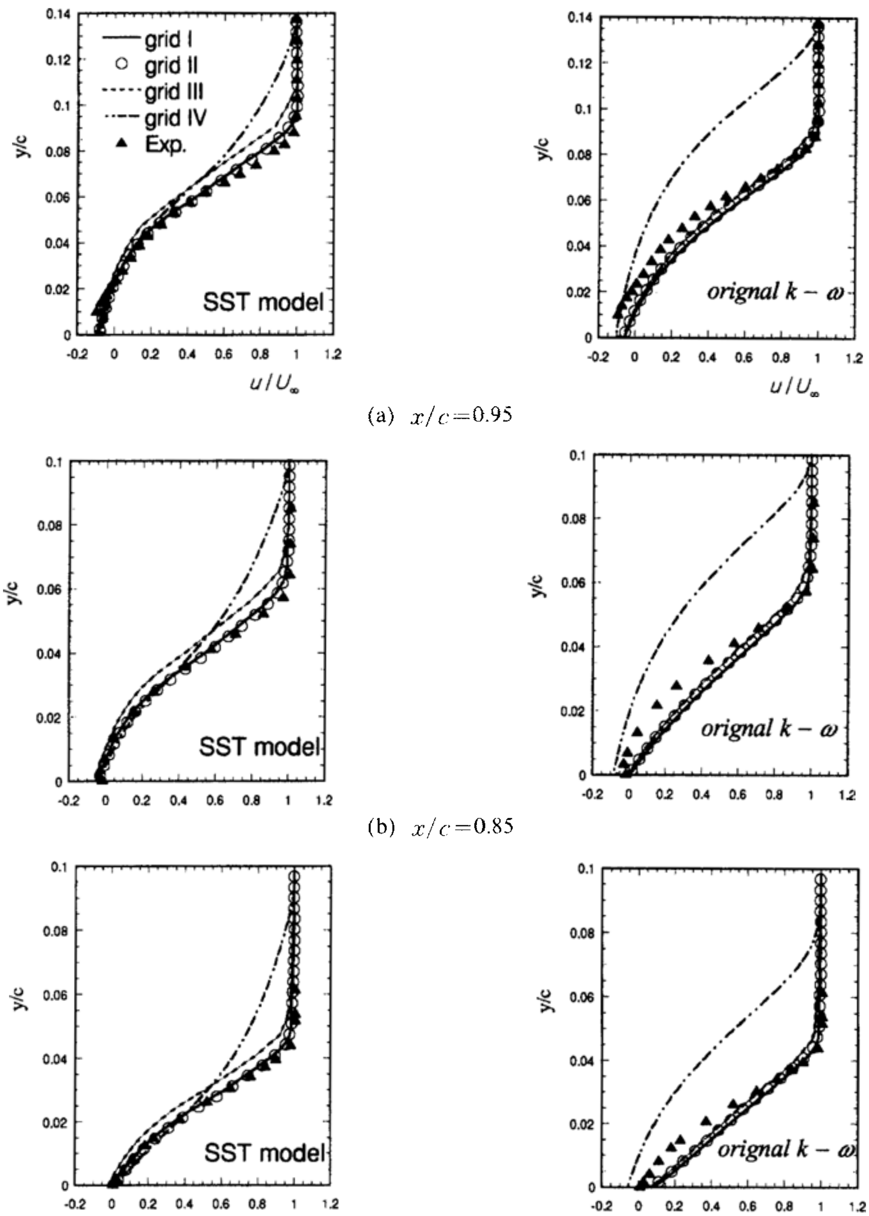


Fig. 1 Unstructured grid around a NACA4412 airfoil (grid I).

region around the airfoil by ALM (advancing layer method), where high aspect ratio of a mesh is required to resolve viscous stress at high Reynolds numbers. Then an unstructured grid is established in the inviscid region away from the airfoil by AFM (advancing front method), where equilateral elements are required and local clustering is important.

3. Numerical Results and Discussions

The meshes used in the present study are listed in Table 1, where the minimum spacing is compared with the chord length which is set to 1. The finest mesh (Grid I) is shown in Fig. 1. The distance from the wall to the nearest point is 10^{-5}



(a) $x/c=0.95$

(b) $x/c=0.85$

(c) $x/c=0.78$

Fig. 2 Upper surface velocity profiles at various x/c positions.

($\Delta y_1^+ \cong 2.0$). To obtain a grid independent solution, the four meshes as listed in Table 1 are tested. Grid IV is the coarsest mesh. We placed more nodes near the surface at the leading edge for Grid III than Grid IV, and even more nodes for Grid II and Grid I. The inlet Reynolds number based on the inlet velocity and the chord length is 4.2×10^6. Figure 2 shows the velocity profiles at various x/c locations, where c is the chord length. It is known that as we adopt denser grids, the present result agrees better with the experimental result of Hastings & Williams (1987), and that Grid I and Grid II give consistent results for both the original $k-\omega$ model and the SST model. Thus, we can argue that they are grid independent solutions for both models. The velocity profiles obtained by using the SST model agree better with the experimental result than the original $k-\omega$ model. From this, we can note that the original $k-\omega$ model should not be used for external flows. In Figure 3, pressure distributions along the surface are shown for both models which exhibit some discrepancies from the experimental result near the leading edge. From the existing studies (Hastings & Williams, 1987; Jansen, 1995), we can argue that such differences are caused by the fact that in the present numerical simulation, wind tunnel wall effect is not considered and the $k-\omega$ model can not simulate precisely the laminar separation and transition phenomenon at the leading edge. We find that the SST model predicts the pressure distribution along the surface satisfactorily except the upper surface of the leading edge, and that grids I and II give identical C_p curves for both models. Figure 4 shows the separation bubble near the trailing edge. Hastings & Williams (1987) reported that boundary layer separation occurs at $x/c\cong0.8$. In the present study, the separation occurs at $x/c\cong$ 0.78 with the SST model and at $x/c\cong0.83$ with the original $k-\omega$ model.

4. Conclusion

A numerical study on the turbulent flow around a NACA4412 airfoil at the maximum lift has been performed using the SUPG finite element method based on the SIMPLE algorithm.

Grid independent solutions are presented for both the original $k-\omega$ and SST models. It has been discussed that the developed code enables the simulation of the turbulent flow with unstructured mesh using apparently less memories than the mixed integrated finite element method.

Fig. 4 Streamline around a NACA4412 airfoil (case 1, SST model).

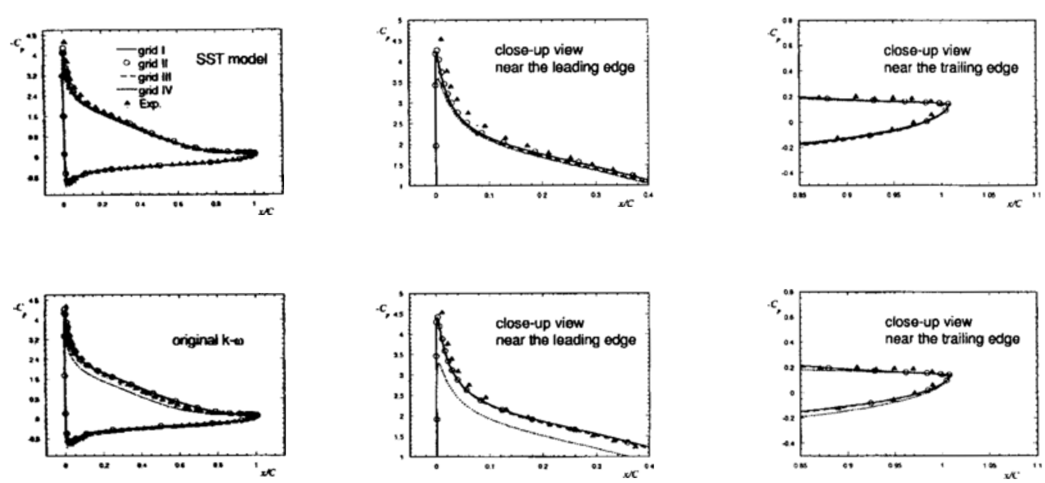


Fig. 3 Pressure distributions along the surface.

It is confirmed that the SST model predicts better the external flow with an adverse pressure gradient than the original $k-\omega$ model.

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